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EE 3413
4/14/16
HW #8

1) $\boxed{\text{First solution}}$

$$G(s) = \frac{10}{s(s+1)}$$

$$\zeta_d = 0.5$$

$$\omega_{nd} = 3$$

$$s_d = -\zeta_d \omega_{nd} \pm \sqrt{1-\zeta_d^2} \cdot \omega_{nd}$$

$$= (-0.5)(3) \pm \sqrt{1-0.25}\,(3)$$

$$= \frac{-3}{2} \pm \frac{\sqrt{3}}{\sqrt{4}}(3)$$

$$= \frac{-3}{2} \pm \frac{3}{2}\sqrt{3}$$

$$S_d = -1.5 \pm j2.59$$

ϕ angle of dificiency

$$\theta = \angle G(s_d) = \angle G(-1.5+j2.59) = 138$$

$$\phi = -180 - (138) = -318 = 42 \text{ deg}$$

$$G_c^{ld}(s) = K \frac{s+z}{s+p}$$

$$z = 1.9432$$
$$p = 4.6458$$

Characteristic

$$1 = |K G(s) \cdot C(s)|$$

$$1 = \left| K \frac{10}{s(s+1)} \cdot \frac{s+1.9432}{s+4.6458} \right|$$

$$K = \left| \frac{s(s+1)(s+4.6458)}{10(s+1.9432)} \right|$$

$$s = -1.5 \pm j2.59$$

$$= \frac{(-1.5+j2.59)(-.5+j2.59)(3.1458+j2.59)}{4.432+25.9j}$$

$$= \left| \frac{-5.3748-31.935j}{4.432+25.9j} \right| \Rightarrow K = 1.213$$

$$G_c^{ld}(s) = K_c \frac{s+z}{s+p}$$

$$K_c = 1.213$$
$$z = 1.9432$$
$$p = 4.6458$$

CLTF: $\dfrac{G_c(s) \cdot G(s)}{1 + G_c(s) G(s) H(s)}$

$$= \frac{\left(1.213 \frac{s+1.9432}{s+4.6458}\right)\left(\frac{10}{s(s+1)}\right)}{1 + 1.213\left(\frac{s+1.9432}{s+4.6458}\right)\left(\frac{10}{s(s+1)}\right)}$$

$$= \frac{1.213(s+1.9432)(10)}{(s+4.6458)s(s+1) + 1.213(s+1.9432)(10)}$$

$$= \frac{12.13s + 23.57}{s^3 + 5.6458s^2 + 16.7758s + 23.57}$$

STATIC-VELOCITY ERROR CONSTANT:

$$K_v = \lim_{s\to 0} s\, G_c(s) G(s)$$

$$= \lim_{s\to 0} s\left[1.213 \cdot \frac{s+1.9432}{s+4.6458} \cdot \frac{10}{s(s+1)}\right]$$

$$= 5.07$$

1) second solution

$$G_c(s) = K_c \frac{s+1}{s+3}$$

Characteristic

$$\left| K_c \frac{\cancel{s+1}}{s+3} \frac{10}{s\cancel{(s+1)}} \right| = 1$$

$$K_c \frac{10}{(s+3)s} = 1$$

$$K_c = \frac{s(s+3)}{10}$$

$$s = -1.5 + j\,2.5981$$

$$= \frac{(-1.5+j\,2.5981)(1.5+j\,2.5981)}{10}$$

$$K_c = \left| \frac{-2.25 - 6.750}{10} \right| = |-.9| = 0.9$$

$$G_c^{ld}(s) = K_c \frac{s+z}{s+p}$$

$$= 0.9 \frac{s+1}{s+3}$$

$$CLTF = \frac{G_c(s)\,G(s)}{1 + G_c(s)\,G(s)H(s)}$$

$$= \frac{0.9 \frac{s+1}{s+3} \frac{10}{s(s+1)}}{1 + 0.9 \frac{s+1}{s+3} \frac{10}{s(s+1)}}$$

$$= \frac{9}{s(s+3)+9}$$

$$= \frac{9}{s^2+3s+9}$$

STATIC-VELOCITY Error Constant

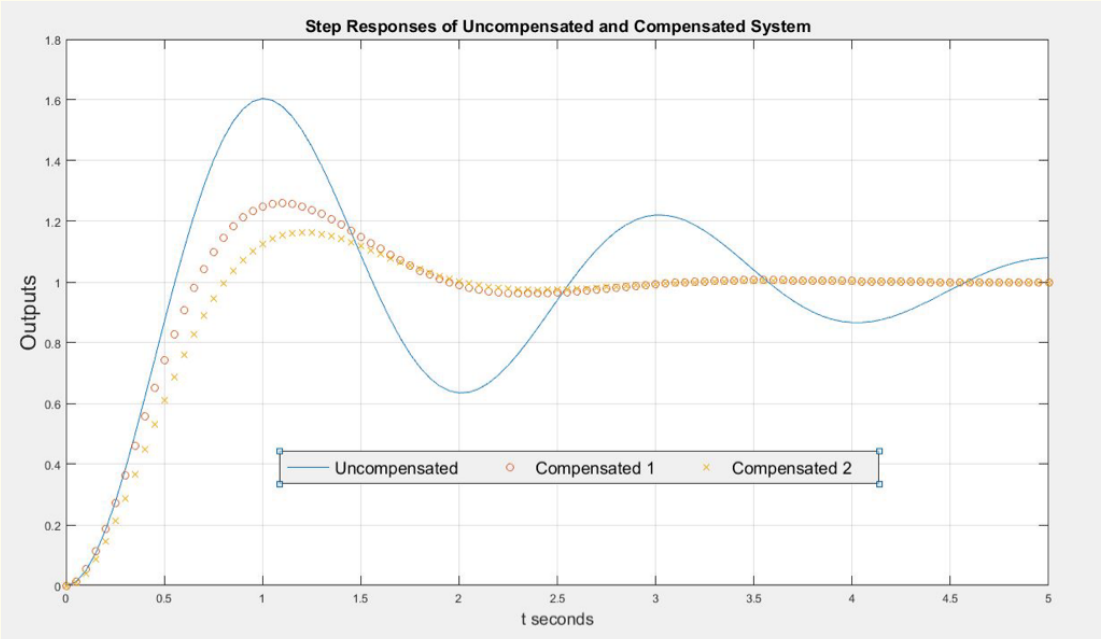
$$K_v = \lim_{s\to 0} s\left(G_c(s)\,G(s)\right)$$

$$= \lim_{s\to 0} s\left[0.9 \frac{\cancel{s+1}}{s+3} \frac{10}{s\cancel{(s+1)}}\right]$$

$$= \lim_{s\to 0} \cancel{s}\left[\frac{9}{(s+3)\cancel{s}}\right]$$

$$= \frac{9}{0+3} = 3$$

1)



Compensated 1 shows a faster rise time and overshoot but a slower settling time compared to compensator 2.

Compensated 2 shows a quicker settling time but a slower rise time and overshoot compared to compensated 1.

2) $G_c(s) = K(Ts+1)$

$G(s) = \dfrac{1}{s(s+2)}$

loop system poles:

$-2 \pm j2$

standard form:

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

CLTF: $\dfrac{G_c(s)G(s)}{1+G_c(s)G(s)H(s)}$

$$= \frac{K(Ts+1)\left(\frac{1}{s(s+2)}\right)}{1+K(Ts+1)\left(\frac{1}{s(s+2)}\right)}$$

$$= \frac{K(Ts+1)}{s(s+2)+K(Ts+1)}$$

$$= \frac{KTs+K}{s^2+(2s+KTs)+K}$$

$$= \frac{-(2+KT) \pm \sqrt{(2+KT)^2-4(1)(K)}}{2}$$

$$= \frac{-(2+KT)}{2} = -2$$

$$= 2+KT = 4$$

$$KT = 2$$

$$\frac{\sqrt{(2+KT)^2-4k}}{2} = 2j$$

$$\left(\frac{\sqrt{(2+KT)^2-4k}}{2}\right)^2 = (2j)^2$$

$$\frac{(2+2)^2-4K}{4} = -4$$

$$16-4k = -16$$

$$k = 8$$

$$T = \frac{2}{8} = \frac{1}{4}$$

CLTF: $\dfrac{KTs+K}{s^2+(2+KT)s+K}$

$$= \frac{8\frac{1}{4}s+8}{s^2+(2+8\frac{1}{4})s+8} \Rightarrow \frac{2s+8}{s^2+4s+8}$$

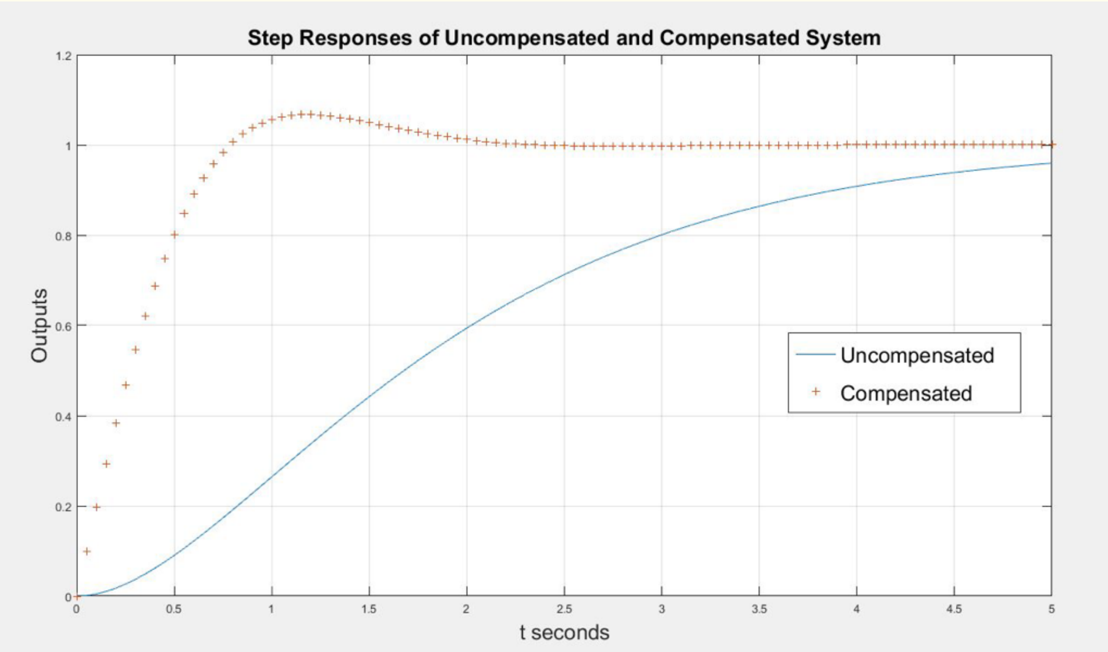
Compensated CLTF

CLTF: $G(s) = \dfrac{1}{s(s+2)}$

$$\Rightarrow \frac{1}{s(s+2)+1}$$

$$= \frac{1}{s^2+2s+1}$$

uncompensated CLTF


Step Responses of Uncompensated and Compensated System
Outputs
t seconds
Uncompensated
Compensated
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1.5
2
2.5
3
3.5
4
4.5
5

3)

Angle of difiency

$$\tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 30$$

$$\theta_1 = 30 + 90 = 120$$

$$\theta_2 = 90$$

$$\phi = -180 - \theta_{Total}$$

$$\phi = -180 - (-210) = 30$$

$$G_c^{ld}(s) = K\,\frac{s+4}{s+7}$$

Characteristic

$$1 = \left| K\, G_c(s)\, G(s) \right|$$

$$K = \left| \frac{s+7}{s+4} \cdot \frac{s(0.5s+1)}{5} \right|$$

$$s = -2 + 3.464j$$

$$K = \left| \frac{-15.603 - 35.76668j}{10 + 15.464j} \right| = \left| \frac{-15.603}{10} \right| = 1.56$$

$$G_c^{ld}(s) = K\,\frac{s+z}{s+p} \Rightarrow 1.56\,\frac{s+4}{s+7}$$

$K_c = 1.56$

$z = 4$

$p = 7$

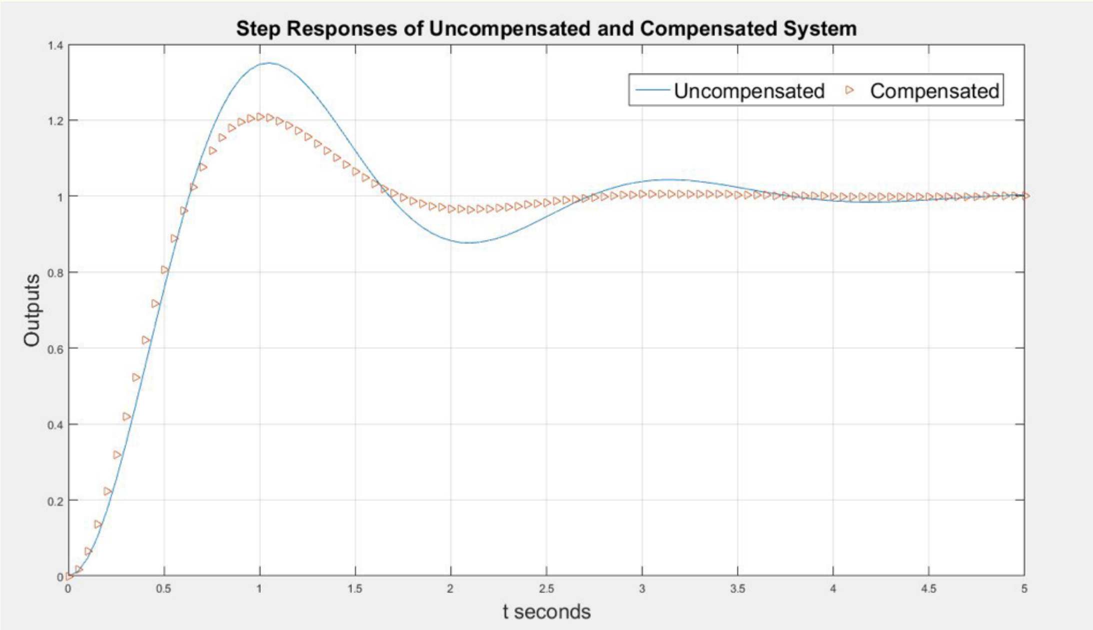
$$G(s) = \frac{5}{s(0.5s+1)}$$

CLTF:
$$\frac{1.56\,\frac{s+4}{s+7}\,\frac{5}{s(0.5s+1)}}{1 + 1.56\,\frac{s+4}{s+7}\,\frac{5}{s(0.5s+1)}}$$

$$= \frac{7.8s + 31.2}{0.5s^3 + 4.5s^2 + 14.8s + 31.2}$$ ← compensated

CLTF:
$$\frac{5}{0.5s^2 + s + 5}$$

$$= \frac{5}{0.5s^2 + s + 5}$$ ← uncompensated



Step Responses of Uncompensated and Compensated System
Uncompensated
Compensated
Outputs
t seconds
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

4) assume:

$$1 + K(\ldots) = K(\ldots)$$

$$\Rightarrow \frac{U(s)}{E(s)} = \frac{K}{1 + K\left(\frac{1}{K_0} \cdot \frac{T_1 s}{1+T_1 s} \cdot \frac{1}{1+T_2 s}\right)}$$

$$= \frac{\cancel{K}}{\cancel{K}\left(\frac{1}{K_0} \cdot \frac{T_1 s}{1+T_1 s} \cdot \frac{1}{1+T_2 s}\right)}$$

$$= \frac{1}{\frac{1}{K_0} \cdot \frac{T_1 s}{1+T_1 s} \cdot \frac{1}{1+T_2 s}}$$

$$= \frac{K_0 (1+T_1 s)(1+T_2 s)}{T_1 s}$$

$$= K_0 \left(\frac{1}{T_1 s} + 1\right)(1 + T_2 s)$$

$$= K_0 \left(\frac{1}{T_1 s} + \underbrace{\frac{T_2 \cancel{s}}{T_1 \cancel{s}} + 1}_{\frac{T_2 + T_1}{T_1}} + T_2 s\right)$$

$$= K_0 \frac{T_2 + T_1}{T_1}\left(1 + \frac{\cancel{T_1}}{T_2 + T_1} \cdot \frac{1}{\cancel{T_1} s} + \frac{T_1}{T_2 + T_1} \cdot T_2 s\right)$$

$$= K_0 \frac{T_2 + T_1}{T_1}\left(1 + \frac{1}{(T_2 + T_1)s} + \frac{T_1 T_2 s}{T_2 + T_1}\right)$$

5)

```
%Homework 8, problem 5
syms s
syms t
syms K
syms a

t = 0:0.01:5; %t in seconds for defining time
for K = 2:1:50; %K parameters
for a = 0.05:0.05:2; %a parameters
    num =[K 2*K*a K*a^2]; %numerator of the CLTF
    den =[1 6 5+K 2*K*a K*a^2]; %denominator of the CLTF
    Y = step(num,den,t); %plot of the CLTF
    X = stepinfo(Y); %list of properties
    O = X.Overshoot; %line used in list of properties
    s = 501; %found with command length(time)
        while Y(s)>0.98 && Y(s)<1.02; %finding the settling time at 2% critieria

        s = s-1;
        end;
    ts = (s-1)*0.01;

if (O)<10 & (O)>2 && ts<3.0 % settling time and overshoot conditions must be met for a
    break;

end
end
if (O)<10 & (O>2) && ts<3.0 % settling time and overshoot conditions must be met for K
   break
end
end
%solutions
plot(t,Y)
xlabel('t seconds')
ylabel('Output')
title('Step Response for K and a')
Optimal_K_and_a = [K;a]
Max_Overshoot_and_Settling_Time = [(O);ts]
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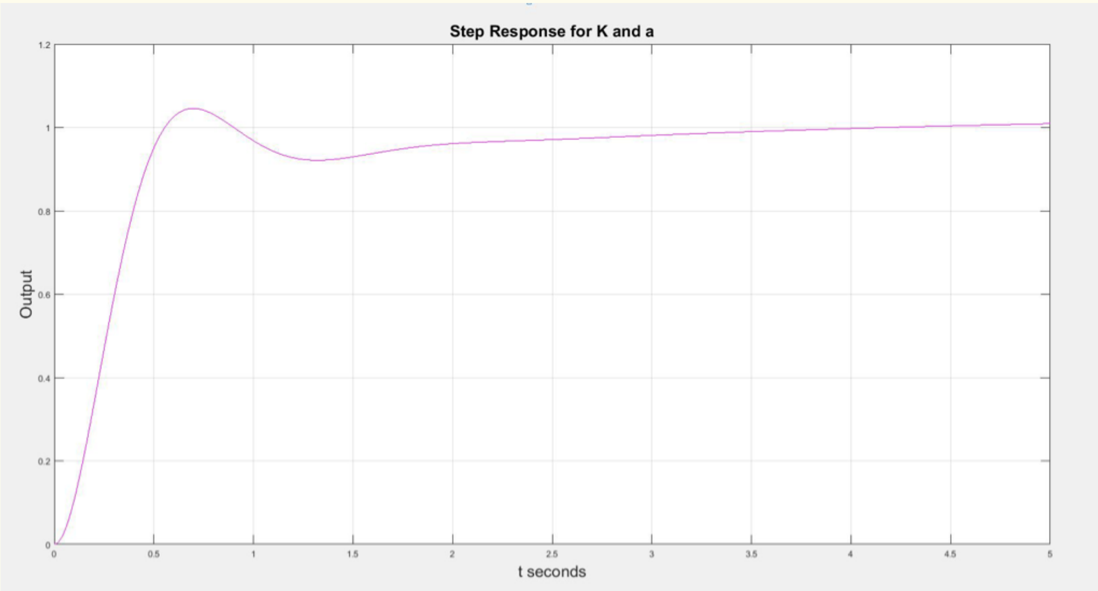
Step Response for K and a
Output
t seconds
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1.5
2
2.5
3
3.5
4
4.5
5