Your Name:

Your Signature:

- **Exam duration:** 1 hour and 20 minutes.
- This exam is closed book, closed notes, closed laptops, closed phones, closed tablets, closed pretty much everything.
- **No calculators** of any kind are allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, **even if your answer is correct.**
- Place a box around your final answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- This exam has 11 pages, plus this cover sheet. Please make sure that your exam is complete, that you read all the exam directions and rules.
- **Question 6 is a bonus question. You do not need to answer it. You should also finish the main exam questions before attempting the bonus one.**

Question Number	Maximum Points	Your Score
1	20	
2	20	
3	25	
4	20	
5	15	
Total	100	
Bonus	15	

1. (20 total points) Find the Laplace transform or the inverse Laplace transform for the following functions. You may use the LT table.

   (a) (5 points) $f_1(t) = e^{2t}\cos(5t) + e^{-3t}\sinh(10t)$. $F_1(s) =$?

   (b) (5 points) $F_2(s) = \dfrac{\sqrt{45}s}{(s^2+16)^2}$. $f_2(t) =$?

(c) (5 points) $f_3(t) = e^{2t}(t^3 + 5t - 2)$. $F_3(s)$=?

(d) (5 points) $F_4(s) = \dfrac{s+1}{(s-2)(s+2)}$. $f_4(t)$ =? You'll have to solve this via partial fraction expansion.

2. (20 total points) The following ODE is given:

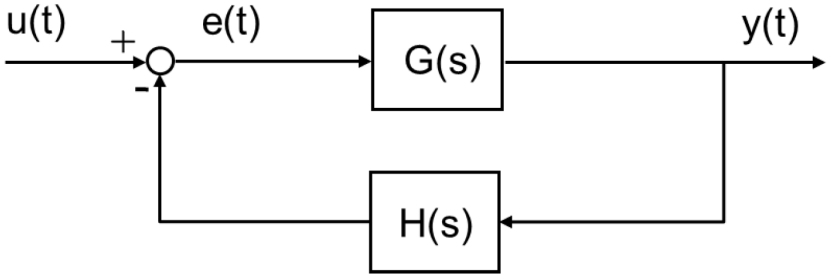
$$y''(t) - y'(t) - 2y(t) = e^{2t}.$$

   (a) (20 points) Given that $y(0) = 0$ and $y'(0) = 1$, find the solution $y(t)$ to the above ODE via Laplace transforms.

3. (25 total points) For the system given in the below figure, assume that:

$$G(s) = \frac{1}{(s-1)(s+3)},$$

$$H(s) = 4.$$



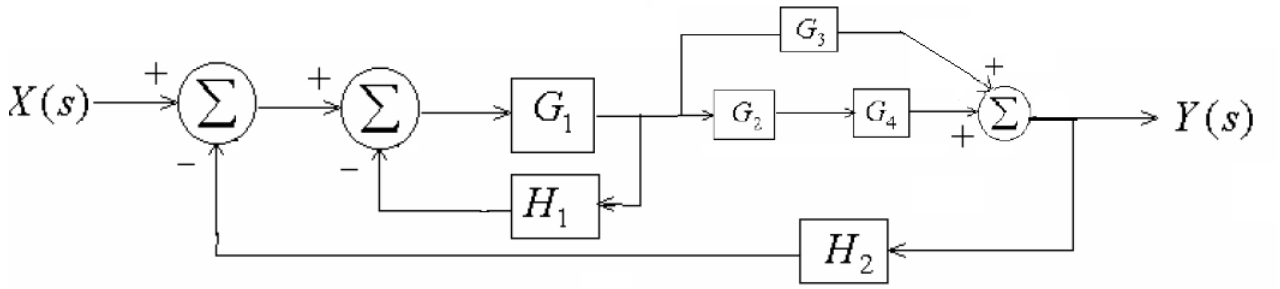
(a) (5 points) Find the transfer function $\frac{Y(s)}{U(s)}$. You can either derive it or just write it down immediately.

(b) (5 points) Find $Y(s)$ if $u(t) = 1$. DO NOT compute $y(t)$.

(c) (5 points) What are the poles of $Y(s)$? Does the final value of $y(t)$ exist (i.e., $y(\infty)$)? If it does, find it via the **final value theorem**. Otherwise, tell me why it doesn't.

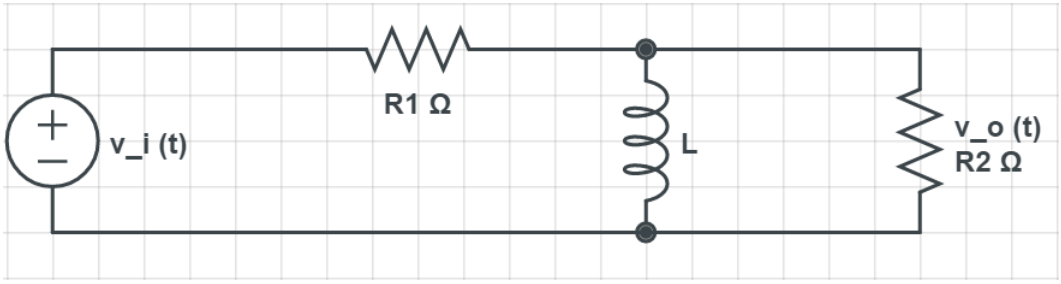
(d) (10 points) Obtain $\dfrac{E(s)}{U(s)}$, then find $E(s)$ for the given $u(t) = 1$. Does the final value of $e(t)$ exist (i.e., $e(\infty)$)? If it does, find it via the **final value theorem**. Otherwise, tell me why it doesn't.

4. (20 total points) You are given the following block diagram.



(a) (20 points) Find $\frac{Y(s)}{X(s)}$ for the above system. Show your work.

5. (15 total points) You are given the following RLC circuit.



(a) (15 points) Derive the transfer function $\frac{V_o(s)}{V_i(s)}$ in terms of R_1, L, and R_2. Show your work. Your transfer function should have the standard form of a transfer function, i.e., polynomials in the numerator and denominator.

6. (15 total points) ***[Bonus Question: Do not answer this before finishing the first five exam questions.]***

   (a) (15 points) Prove the initial value theorem:

$$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s).$$

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			