The objective of this homework is to test your understanding of the content of Module 5. Due date of the homework is: **Thursday**, **February 25th**, **2016**, **@ noon**.

You have to upload a scanned version of your solutions on Blackboard. If you don't have a scanner around you, you can use Cam Scanner—a mobile app that scans images in a neat way, as if they're scanned through a copier. Here's the link for Cam Scanner: https://www.camscanner.com/user/download.

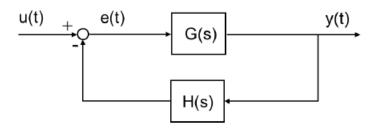


Figure 1: Feedback control system.

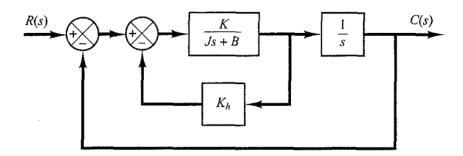


Figure 2: Servo system.

1. For a standard second order system given by this transfer function:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2},$$

where  $\zeta = 0.6$  and  $\omega_n = 5$ . Answer the following questions.

- (a) Find the: rise time, peak time, maximum overshoot, and settling time (the two criterion we discussed in class) if the system input is a unit step function.
- (b) Show a plot of how  $t_r$ ,  $t_p$ , and  $M_p$  all vary with respect to different values of  $\zeta$  and  $\omega_n$ . Ideally, you should do that on MATLAB.
- 2. For the system shown in Figure 1, assume that  $G(s) = \frac{-K}{s+10}$  and H(s) = 1. Answer the following questions:
  - (a) Find the closed-loop transfer function Y(s)/U(s) and its pole (or poles).

- (b) What is the range of the constant *K* so that the closed-loop system is stable?
- (c) Suppose K = 5. What is the time constant of the closed-loop transfer function (as a first order system)?
- (d) What is the steady-state tracking error  $e(\infty) = u(\infty) y(\infty)$  under the input a unit step input u(t)?
- 3. For the system given in Figure 2, answer the following questions.
  - (a) Obtain the transfer function C(s)/R(s) in terms of constants K, J, B,  $K_h$ , and then write this system as a standard second order system as the transfer function given in Problem 1.
  - (b) Determine the values of gain K and  $K_h$  so that  $M_p$  (the maximum overshoot) for a unit step response is equal to 0.2, and  $t_p$  (the peak time) is 1 second. Assume that J = 1 and B = 1.
  - (c) With the above, now-obtained values for K and  $K_h$ , obtain the rise-time and settling time.