QUIZ # 2

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Given the following LTI dynamical system:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x_{\text{initial}} = x_{t_0} \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, x(t_0) = x(1) = [0 \; 1 \; 1]^\top.$$

Recall that the closed-form to the above differential equation for any time-varying control input is given by:

$$x(t) = e^{A(t-t_0)}x_{t_0} + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)\, d\tau.$$

1. Is A nilpotent of order 2? (i.e., is $A^2 = 0$?)

2. Determine $e^{At}, e^{A(t-\tau)}, e^{A(t-t_0)}$, $t_0 = 1$. Recall that

$$e^{At} = \sum_{i=0}^{\infty} \frac{(At)^i}{i!} = \boldsymbol{I}_n + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \cdots$$

3. If $u(t) = 0$, determine $x(t)$ (or the zero-input state state-response) given the provided initial conditions.

4. If $x(t_0) = x(1) = 0$ and $u(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} 1^+(t)$, determine $x(t)$ (or the zero-state, state-response).

5. Determine $y(t)$ if $u(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} 1^{+}(t)$ and $x(t_0) = x(1) = [0\ 1\ 1]^{\top}$.