THE UNIVERSITY OF TEXAS AT SAN ANTONIO
EE 5243
INTRODUCTION TO CYBER-PHYSICAL SYSTEMS

QUIZ # 5 SOLUTION

Ahmad F. Taha

September 21, 2015

Find a solution (or solutions) that satisfies the KKT conditions for the following optimization problem:

$$\underset{x}{\text{minimize}} \quad f(x) = 2x_1 + x_2 \tag{1}$$

$$\text{subject to} \quad h(x) = x_1 + x_2 - 1 = 0 \tag{2}$$

$$g(x) = x_1 + 2x_2 - 2 \leq 0 \tag{3}$$

The KKT conditions are given by:

1. $\nabla_x \mathcal{L}(x^*, \lambda^*, \mu^*) = \nabla_x f(x) + \lambda^* \nabla_x h(x^*) + \mu^* \nabla_x g(x^*) = 0$
2. $\mu^* \geq 0$
3. $\mu^* g(x^*) = 0$
4. $g(x^*) \leq 0$
5. $h(x^*) = 0$

Solution: Conditions are as follows (we drop the * for brevity):

1. $2 + \lambda + \mu = 0$
2. $1 + \lambda + 2\mu = 0$
3. $x_1 + x_2 - 1 = 0$
4. $\mu(x_1 + 2x_2 - 2) = 0$
5. $x_1 + 2x_2 - 2 \leq 0$

Solving 1. and 2., we obtain: $\lambda^* = -3, \mu^* = 1$. From 3. and 4., we get: $x_1^* = 0, x_2^* = 1$. This solution clearly satisfies condition 5.

Hence, $\mathbf{x}^* = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ satisfies the KKT conditions and is a candidate for being a minimizer for the given optimization problem.