QUIZ # 7

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- Hamiltonian:

$$\mathcal{H}(x, u, \lambda^*(x,t), t) = g(x, u, t) + \lambda^*(x,t) f(x, u, t)$$

- Value function properties:
  1. $V_x(x,t) = \frac{\partial V}{\partial x} = \lambda^*(x,t)$
  2. $-V_t(x,t) = -\frac{\partial V}{\partial t} = \min_{u \in \mathcal{U}} \mathcal{H}(x, u, \lambda^*(x,t), t) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^\top$

- The **HJB Equation:**

$$\boxed{-V_t^*(x,t) = -\frac{\partial V}{\partial t} = \min_{u \in \mathcal{U}} \mathcal{H}(x, u, \lambda^*(x,t), t) = \left(\frac{\partial \mathcal{H}}{\partial x}\right)^\top}$$

For this optimal control problem,

$$\begin{aligned} \text{minimize } J \quad &= \quad \frac{1}{2} x_{t_f}^\top H x_{t_f} + \frac{1}{2} \int_{t_0}^{t_f} \left[x(t)^\top Q(t) x(t) + u(t)^\top R(t) u(t) \right] dt \\ \text{subject to} \quad & \quad \dot{x}(t) = A(t)x(t) + B(t)u(t), \end{aligned}$$

answer the following questions:

1. Construct the **Hamiltonian**.
2. Find the optimal $u^*(t)$ in terms of $\lambda^*(x,t)$.
3. Write the **Hamiltonian** in terms of $u^*(t)$.
4. Apply the value function properties (above) for this candidate value function:

$$V^*(x,t) = \frac{1}{2} x^\top(t) P(t) x(t), \; P(t) = P^\top(t)$$

5. Based on the given, derive the Differential Riccati Equation that relates $\dot{P}(t)$ with $P(t)$, and explain how can $u^*(t)$ be obtained.