Given the following plant dynamics:

$$\dot{x}_p = A_p x_p + B_p^{(1)} u_1 + B_p^{(2)} u_2$$
  
 $y = C_p x_p$ ,  $x_p(0)$  not given

where  $u_2(t)$  is the unknown input vector. The system consists of n states,  $m_1$  known inputs,  $m_2$  unknown inputs, and p measurable outputs. We want to design a dynamic unknown input observer (UIO) which takes the following form:

$$\dot{x}_c = A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1,$$
  
 $\dot{x}_p = x_c + M y,$ 

The UIO is motivated by writing  $x_p$  as:

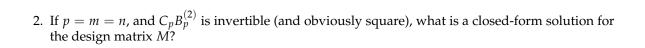
$$x_p = (I - MC_p)x_p + MC_px_p = x_c + My.$$

1. Assume that the updated  $x_c$  takes the following form:

$$x_c = (I - MC_p)x_p.$$

- (a) Find  $\dot{x}_c = A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1$ ,, where  $A_c, B_c^{(1)}, B_c^{(2)}$  are matrices that you should determine, assuming that the unknown input vector is nullified and a convergence term is added to  $x_c$ , as discussed in class. Note that  $\hat{x}_p = x_c + My$ ;
- (b) Derive the matrix equality that guarantees the nullification of  $u_2(t)$ .

Precisely, you should find  $A_c$ ,  $B_c^{(1)}$ ,  $B_c^{(2)}$  in terms of  $A_p$ ,  $B_p^{(1)}$ ,  $C_p$ , M, L.



3. Derive the estimation error dynamics.